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Calculation of Radiative Corrections to E1 matrix elements in the Neutral Alkalies

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Abstract

Radiative corrections to E1 matrix elements for $ns - np$ transitions in the alkali metal atoms lithium through francium are evaluated. They are found to be small for the lighter alkalis but significantly larger for the heavier alkalis, and in the case of cesium much larger than the experimental accuracy. The relation of the matrix element calculation to a recent decay rate calculation for hydrogenic ions is discussed, and application of the method to parity nonconservation in cesium is described.

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I. INTRODUCTION

High accuracy measurements of atomic lifetimes are difficult, with only a few examples of precisions well under one percent known. For this reason, consideration of radiative corrections to lifetimes is usually not necessary. Until recently the most prominent exception to this situation was the decay rates of orthopositronium and parapositronium, where determinations of accuracy 180 ppm [1] and 215 ppm [2] respectively have been made. In these exotic atoms the radiative corrections start in order α with large coefficients, and the leading radiative correction are clearly visible, both because in these two-body atoms the wave function is known analytically and because quantum electrodynamic (QED) corrections start in order α with large coefficients, giving contributions of 2.1 percent and 0.5 percent respectively.

In recent years, however, a new approach has been developed that exploits the fact that the dipole-dipole potential between two alkali atoms, which goes as C_3/R^3 , can be accurately measured, and C_3 is proportional to the lifetimes of np states of the atoms. This has allowed the determination of the lifetime of the $2p_{1/2}$ state of lithium as 27.102(7) ns [3], the lifetime of the $3p_{3/2}$ state of sodium as 16.230(16) ns [4], the lifetimes of the $4p_{1/2}$ and $4p_{3/2}$ states of potassium as 26.69(5) ns and 26.34(5) ns [5], the lifetimes of the $5p_{1/2}$ and $5p_{3/2}$ states of rubidium as 27.75(8) ns and 26.25(8) ns [6], and the lifetimes of the $6p_{1/2}$ and $6p_{3/2}$ states of cesium as 34.88(2) ns and 30.462(3) ns [7]. (References to other experimental determinations of lifetimes, some of which are of higher accuracy, can be found in the above references. We note also that the lifetimes of the francium $7p_{1/2}$ and $7p_{3/2}$ states have been measured as 29.45(11) ns and 21.02(11) ns using a different technique [8].) These high accuracies, which for matrix elements correspond at the best to 50 ppm, now make the calculation of radiative corrections of interest, even though unlike the case of positronium the corrections to E1 matrix elements are known to enter, for hydrogenic ions [9], in order $\alpha(Z\alpha)^2$.

There are two other reasons for carrying out calculations of these radiative corrections. Firstly, this is a relatively unexplored region of QED. It has only been very recently that the first full calculations of one-loop radiative corrections to the decay rate of the hydrogen isoelectronic sequence have been carried out [9]. That calculation was done by considering the imaginary part of the two-loop Lamb shift, which is equivalent to calculating the shift in

the lifetime. Here we adopt a different method, and instead calculate radiative corrections to the associated transition matrix element. The introduction of this technique requires a nontrivial modification of the previous formalism, and also provides a check of that method.

The second additional reason for calculating radiative corrections to transition matrix elements in the alkalis is the interest in parity nonconservation (PNC) in cesium [10]. Recently a large binding correction to the Z boson-electron vertex radiative correction has been found that has significant implications for the standard model. The lowest-order radiative correction, $-\alpha/2\pi$, has been shown [11–13] to be enhanced through binding corrections by almost an order of magnitude. However, the actual radiative correction is to the E1 matrix element of a 6s electron to a 7s electron, with an opposite parity component present in one of these electrons induced by Z exchange with the nucleus. The full radiative correction calculation needed then involves the evaluation of the diagrams in Fig. 1, of which diagram 1c, where the radiative correction is on the photon rather than the Z vertex, is of the kind to be treated here. Thus the calculations on cesium that will be presented here will be of use for this larger scale task.

We note that the methods used in this paper have previously been applied to calculating the Lamb shift [14] and the radiative correction to hyperfine splitting [15] for the ground states of the alkalis, where QED effects larger than experimental accuracy were found. However, the calculations remain untested because of the relative inaccuracy of many-body methods (with the exception of lithium, where highly accurate variational methods are available [16]). We will see that the same situation is present for alkali lifetimes, and thus the present work provides yet another impetus to many-body theory to reach the accuracies presently of interest for both radiative corrections and experiment.

In the next section, the generalization of the method used for the previous calculations of radiative corrections in the alkalis will be laid out. Of particular interest are certain issues related to the fact that the Gell-Mann-Low formalism for energies used in the past work has to be changed because we are now instead dealing with matrix elements. In Section III, the technique is applied to hydrogenic ions to compare with previous work, and in Section IV the main results of the paper are presented. We conclude with a discussion of how this approach can be made more accurate and how it can be generalized for application to the calculation of radiative corrections to cesium PNC.

II. FORMALISM

When one is interested in calculating energy shifts in atoms, Sucher's generalization [17] of the Gell-Mann-Low formalism provides a systematic way to derive them from the S-matrix through the formula

$$\Delta E = \lim_{\epsilon \rightarrow 0, \lambda \rightarrow 1} \frac{i\epsilon}{2} \frac{1}{S_{\epsilon, \lambda}} \frac{\partial S_{\epsilon, \lambda}}{\partial \lambda}, \quad (1)$$

where $S_{\epsilon, \lambda}$ is the S-matrix with the interaction Hamiltonian $H_I(t)$ multiplied by the factor

$$\lambda e^{-\epsilon|t|}. \quad (2)$$

The overall factor of ϵ is compensated by the fact that the S-matrix diverges as $1/\epsilon$. In higher orders, factors of $1/\epsilon^2$ are encountered in the numerator that are canceled by $1/\epsilon$ terms coming from expanding the denominator. When considering matrix elements the factor λ , which accounts for a combinatorial factor specific to the Gell-Mann-Low formalism, will not be used, but we continue to use the exponential damping factor ϵ , which will lead to the frequent occurrence of what is effectively the delta function,

$$D_\epsilon(x) = \frac{1}{\pi} \frac{\epsilon}{x^2 + \epsilon^2}. \quad (3)$$

We replace the time-independent perturbation of Ref. [18] with a time dependent Hamiltonian in the length-gauge form appropriate for describing the absorption of a photon in a electromagnetic field of strength E_0 linearly polarized in the z direction,

$$H_{\text{LG}} = eE_0 \int d^3x \psi^\dagger(\vec{x}, t) \vec{x} \cdot \hat{z} e^{-i\omega t} \psi(\vec{x}, t) a(\vec{k}, \hat{z}). \quad (4)$$

We have made the dipole approximation, a good approximation for the neutral alkalis, so the momentum of the initial photon, \vec{k} , plays no role in the following. In addition we suppress the factor eE_0 in the following. We consider the matrix element of this Hamiltonian between an initial state v taken to be a ns state ($2s$ for lithium, $3s$ for sodium, etc.) along with a photon with energy ω , and a final state w taken to be a $np_{1/2}$ or $np_{3/2}$ state with the same principal quantum number n . It is important to keep the photon energy ω distinct from the resonance energy $\omega_0 \equiv \epsilon_w - \epsilon_v$, and in particular the limit $\epsilon \rightarrow 0$ is always understood to be taken before the limit $\omega \rightarrow \omega_0$. In lowest order, the S-matrix is then

$$S = -2\pi i D_\epsilon(\omega - \omega_0) z_{wv}, \quad (5)$$

and following the convention just mentioned, this becomes the usual

$$S = [-2\pi i \delta(\omega - \omega_0)] z_{wv}. \quad (6)$$

In the following, we shall suppress the factor in square brackets. In Table I, we present results for the dipole matrix element r_{wv} , with

$$r_{wv} = \int_0^\infty dr \, r \left[g_w(r) g_v(r) + f_w(r) f_v(r) \right], \quad (7)$$

from which z_{wv} can be obtained by either multiplying by a factor of $(1/3)(-1)^{j_w - m_v}$ or $\sqrt{2}/3$ for the $ns_{1/2} - np_{1/2}$ and $ns_{1/2} - np_{3/2}$ transitions, respectively. These are lowest-order results obtained using the local Kohn-Sham potential, a description of which can be found in Ref. [14]. Issues involved in correcting these results with many-body methods will be addressed in the concluding section, but here we will concentrate on radiative corrections. In evaluating these we will work in terms of the ratio R_{wv} defined through

$$\delta z_{wv} \equiv \frac{\alpha}{\pi} z_{wv} R_{wv}, \quad (8)$$

such that

$$z_{wv} + \delta z_{wv} = z_{wv} \left(1 + \frac{\alpha}{\pi} R_{wv} \right). \quad (9)$$

At this point we also define the frequently occurring self-energy operator

$$\Sigma_{ij}(\epsilon) = -4\pi i \alpha \int d^3x \int d^3y \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 + i\delta} \bar{\psi}_i(\vec{x}) \gamma_\mu S_F(\vec{x}, \vec{y}; \epsilon - k_0) \gamma^\mu \psi_j(\vec{y}), \quad (10)$$

in terms of which the lowest-order self-energy part of the Lamb shift of a state v , treated in Ref. [14], is simply $\Sigma_{vv}(\epsilon_v)$.

Most of the discussion of radiative corrections to z_{wv} is very similar to the treatment using the Gell-Mann-Low formalism given in Ref. [18], except here we pull out a factor of $-2\pi i \delta(\omega - \omega_0)$ as opposed to isolating $1/\epsilon$ terms. As described in more detail in that paper, three diagrams shown in Fig. 2 contribute to the radiative correction to the matrix element. The S-matrix associated with the vertex (V) diagram of Fig. 2b is given by

$$\begin{aligned} S_V = & -32\pi^4 \alpha \int d^3x \int d^3y \int d^3z \int \frac{d^n k}{(2\pi)^n} \frac{e^{i\vec{k} \cdot (\vec{x} - \vec{z})}}{k^2 + i\delta} \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \bar{\psi}_w(\vec{x}) \gamma_\mu S_F(\vec{x}, \vec{y}; E_1) \vec{y} \cdot \hat{z} \gamma_0 \\ & \times S_F(\vec{y}, \vec{z}; E_2) \gamma^\mu \psi_v(\vec{z}) D_\epsilon(E_1 + k_0 - \epsilon_w) D_\epsilon(E_2 + k_0 - \epsilon_v) D_\epsilon(E_2 - E_1 + \omega). \end{aligned} \quad (11)$$

One can make the substitutions $E_1 \rightarrow \epsilon_w - k_0$ and $E_2 \rightarrow \epsilon_v - k_0$ in the electron propagators, which then allows the E_1 and E_2 integrations to be carried out,

$$\begin{aligned} & \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} D_\epsilon(E_1 + k_0 - \epsilon_w) D_\epsilon(E_2 + k_0 - \epsilon_v) D_\epsilon(E_2 - E_1 - \omega) \\ &= \frac{3\epsilon}{4\pi^3} \frac{1}{(\omega - \omega_0)^2 + 9\epsilon^2} \\ &\rightarrow \frac{\delta(\omega - \omega_0)}{4\pi^2}. \end{aligned} \quad (12)$$

We note that in energy calculations where the factor $\omega - \omega_0$ vanishes, a factor $1/3$ results that is canceled because a derivative with respect to the factor λ present in the energy formula acts on a factor λ^3 . Here the factor $1/3$ is not present for a different reason, that being the fact that the effective infinitesimal factor used to obtain the energy conserving delta function is 3ϵ . We then find, after pulling out the factors mentioned above, a vertex contribution of

$$\begin{aligned} \delta z_{wv}(V) &= -4\pi i\alpha \int d^3x \int d^3y \int d^3z \int \frac{d^n k}{(2\pi)^n} \frac{e^{i\vec{k}\cdot(\vec{x}-\vec{z})}}{k^2 + i\delta} \bar{\psi}_w(\vec{x}) \gamma_\mu S_F(\vec{x}, \vec{y}; \epsilon_w - k_0) \vec{y} \cdot \hat{z} \gamma_0 \\ &\quad \times S_F(\vec{y}, \vec{z}; \epsilon_v - k_0) \gamma^\mu \psi_v(\vec{z}). \end{aligned} \quad (13)$$

This expression has both ultraviolet divergences and reference state singularities that cancel with the side graphs of Figs. 2a and 2c discussed below. The ultraviolet divergence is isolated analytically by replacing both bound state propagators with free propagators, which gives an ultraviolet divergent term along with a finite remainder we tabulate as $R_{wv}(V; 00)$ in the first row of Table II. (We note at this point that we present results only for $ns - np_{1/2}$ transitions. Those for $ns - np_{3/2}$ transitions should not be too different for neutral systems considered here, as radiative corrections to the lifetimes of the $2p_{1/2}$ and $2p_{3/2}$ states are essentially the same for low- Z hydrogenic ions [9].) We then form the ultraviolet finite difference of $\delta z_{wv}(V)$ and $\delta z_{wv}(V; 00)$ and evaluate it in coordinate space. A Wick rotation $k_0 \rightarrow i\omega$ is carried out, which passes poles: the separate contributions are tabulated as $R_{wv}(V; i\omega)$ and $R_{wv}(V; \text{Poles})$ in the second and third rows of Table II. The reference state singularity mentioned above is present in the first term, and is regulated through the replacement $\epsilon_v \rightarrow \epsilon_v(1 - i\delta)$ and $\epsilon_w \rightarrow \epsilon_w(1 - i\delta)$ with δ typically chosen to be 10^{-6} .

The treatment of the side graphs also differs from the previous energy approach. The starting expression for the “side-right” (SR) diagram of Fig. 2a is

$$S_{\text{SR}} = -32\pi^4 \alpha \int d^3x \int d^3y \int d^3z \int \frac{d^n k}{(2\pi)^n} \frac{e^{i\vec{k}\cdot(\vec{y}-\vec{z})}}{k^2 + i\delta} \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \bar{\psi}_w(\vec{x}) \gamma_0 \vec{x} \cdot \hat{z} S_F(\vec{x}, \vec{y}; E_1) \gamma_\mu$$

$$\times S_F(\vec{y}, \vec{z}; E_2) \gamma^\mu \psi_v(\vec{z}) D_\epsilon(E_1 + \omega - \epsilon_w) D_\epsilon(E_2 + k_0 - \epsilon_v) D_\epsilon(E_2 + k_0 - E_1). \quad (14)$$

Replacing the first electron propagator with a spectral representation gives

$$\begin{aligned} S_{\text{SR}} = & -8i\pi^3 \alpha \sum_m \int d^3x \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \frac{\bar{\psi}_w(\vec{x}) \gamma_0 \vec{x} \cdot \hat{z} \psi_m(\vec{x})}{E_1 - \epsilon_m(1 - i\delta)} \Sigma_{mv}(E_2) \\ & \times D_\epsilon(E_1 + \omega - \epsilon_w) D_\epsilon(E_2 + k_0 - \epsilon_v) D_\epsilon(E_2 + k_0 - E_1). \end{aligned} \quad (15)$$

The D_ϵ functions emphasize $E_1 = \epsilon_v$. If the state v is excluded in the sum over states m , the same kind of manipulations applied to the vertex graph allow one to determine a “perturbed orbital” (PO) contribution from the SR diagram of

$$\delta z_{wv}(\text{PO}; ns) = \int d^3x \sum_{m \neq v} \frac{\bar{\psi}_w(\vec{x}) \gamma_0 \vec{x} \cdot \hat{z} \psi_m(\vec{x})}{\epsilon_v - \epsilon_m + i\delta} \Sigma_{mv}(\epsilon_v). \quad (16)$$

The notation (PO; ns) refers to the fact that this contribution is essentially the self-energy $\Sigma_{\tilde{v}v}(\epsilon_v)$, where \tilde{v} is a perturbation of the ns state. A similar contribution, $\delta z_{wv}(\text{PO}; np)$ arises from the side-left diagram, and is so designated because it is $\Sigma_{w\tilde{w}}(\epsilon_w)$. Its specific value is

$$\delta z_{wv}(\text{PO}; np) = \int d^3x \sum_{m \neq w} \Sigma_{wm}(\epsilon_w) \frac{\bar{\psi}_m(\vec{x}) \gamma_0 \vec{x} \cdot \hat{z} \psi_v(\vec{x})}{\epsilon_w - \epsilon_m + i\delta}. \quad (17)$$

The case when $m = v$ requires more care. We need to make a Taylor expansion of the electron propagator in the self-energy function around the point $E_2 = \epsilon_v - k_0$,

$$S_F(\vec{y}, \vec{z}; E_2) = S_F(\vec{y}, \vec{z}; \epsilon_v - k_0) + (E_2 - \epsilon_v + k_0) S'_F(\vec{y}, \vec{z}; \epsilon_v - k_0) + \dots \quad (18)$$

The first term of the expansion is highly divergent, involving the integral

$$\begin{aligned} & \int \frac{dE_1}{2\pi} \int \frac{dE_2}{2\pi} \frac{1}{E_1 - \epsilon_v + i\delta} D_\epsilon(E_1 + \omega - \epsilon_w) D_\epsilon(E_2 + k_0 - \epsilon_v) D_\epsilon(E_2 + k_0 - E_1) \\ = & \frac{1}{8i\pi^3 [(\omega - \omega_0)^2 + \epsilon^2]} - \epsilon \left\{ \frac{\omega - \omega_0}{\pi^3 [(\omega - \omega_0)^2 + 9\epsilon^2] [(\omega - \omega_0)^2 + \epsilon^2]} \right\}. \end{aligned} \quad (19)$$

Because we take the limit $\epsilon \rightarrow 0$ before $\omega \rightarrow \omega_0$ the second term can be dropped, leaving the divergent expression

$$\delta z_{wv}(\text{Div}) = -\frac{i}{2\epsilon} z_{wv} \Sigma_{vv}(\epsilon_v) \quad (20)$$

In the energy formalism this corresponds to a $1/\epsilon^2$ term that is canceled by a term in the denominator of Eq. (1). Here it does not cancel, but instead forms the second term of the Taylor expansion of the phase factor

$$e^{-i[\Sigma_{vv}(\epsilon_v) + \Sigma_{wv}(\epsilon_w)]/2\epsilon} \quad (21)$$

multiplying z_{wv} , where we have now included the effect of the side-left diagram. Consideration of higher-order diagrams with two self-energies and a photon interaction show that even more divergent terms going as, for example, $\Sigma_{vv}^2(\epsilon_v)/\epsilon^2$ are present that continue the Taylor expansion. Thus this divergent term is not directly canceled as in the energy formalism but instead can be ignored because it enters only as a phase [19]. However, the second term in Eq. (18) does contribute a finite amount to the scattering amplitude,

$$\delta z_{wv}(\text{D}) = \frac{1}{2} z_{wv} [\Sigma'_{vv}(\epsilon_v) + \Sigma'_{ww}(\epsilon_w)], \quad (22)$$

where we have included a similar term arising from the side-left diagram. The treatment of these derivative (D) terms follows that of the vertex, but in this case we simply sum the finite part of the free propagator term, the Wick rotated term, and the terms in which poles are encircled and present the results as $R_{wv}(\text{D})$ in the fourth row of Table II. The ultraviolet infinite part of the derivative terms cancels a similar term from the vertex exactly, and the reference states are allowed to cancel numerically. Finally, the two perturbed orbital terms are listed as $R_{wv}(\text{PO}; ns)$ and $R_{wv}(\text{PO}; np)$ in the fifth and sixth rows of Table II. The sums of all contributions give the radiative correction results R_{wv} shown in the last row of Table II.

III. HYDROGENIC IONS

In a previous paper [9], the imaginary part of the two-loop Lamb shift was used to calculate radiative corrections to the $2p_{1/2}$ and $2p_{3/2}$ lifetimes of hydrogenic ions. In this section, we redo the $2p_{1/2}$ correction for one of these ions using the present matrix element formalism as a check of both approaches and to show the role of radiative energy shifts in lifetime corrections. The $2p_{3/2}$ case included M1 decays, and so cannot be used for a direct comparison.

In Ref. [9], the radiative correction to the decay rate Γ is defined in terms of the function $R(Z\alpha)$ by

$$\Gamma = \Gamma_0 \left[1 + \frac{\alpha}{\pi} R(Z\alpha) \right]. \quad (23)$$

An important feature of these corrections is that both radiative corrections to energies and radiative corrections to matrix elements are of similar importance. As E1 decay rates in the length gauge are the product of the cube of the energy difference ω_0 and the square

of the transition matrix element z_{wv} , one component of $R(Z\alpha)$ which arises from energy corrections should be given by

$$3 \frac{\delta E}{E} = \frac{\alpha}{\pi} (Z\alpha)^2 \left[-\frac{32}{3} \ln(Z\alpha)^{-2} + 22.815 \right] \quad (24)$$

where we have used the standard values of the leading $Z\alpha$ -expansion contributions to the $1s$ and $2p_{1/2}$ Lamb shifts. At $Z = 5$, this contributes -0.064 to $R(Z\alpha)$. However, the actual value of this function calculated in Ref. [9] was -0.014 , so that a substantial positive contribution of about 0.050 must come from the shift in the matrix element. We have carried out this calculation using the techniques described above and find a result of 0.025 for the function R_{wv} defined in Eq. (8). This result is in perfect agreement with the expected value, as it has to be doubled to account for the fact that the matrix element enters as a square in decay rate calculations. But while this result shows the consistency between the present matrix-element and the former decay-rate approaches, there is no improvement in the numerical accuracy using our present method for these radiative correction calculations.

IV. RESULTS AND DISCUSSION

As found with our previous work on E1 matrix elements for hydrogenic ions [9], there is a high degree of cancellation between contributing terms shown in the first six rows of Table II. This is due to the fact that radiative corrections to decay rates enter in order $\alpha(Z\alpha)^2$. In fact, if one puts the perturbed orbital contributions aside, the cancellation is almost complete. As a result, the radiative correction is dominated by the (PO, ns) terms, which are the larger of the two perturbed orbital terms. Since it is a standard practice with the alkalis to derive matrix elements from observed lifetimes by dividing out the cube of the experimental energy differences, we do not need to consider radiative corrections to energies here, as they are automatically excluded in empirically extracted matrix elements.

The experimental accuracies for the alkali matrix elements are half the accuracies of the lifetimes quoted in the introduction, specifically being 130 ppm, 490ppm, 940ppm, 1440ppm, 50 ppm, and 1870 ppm for lithium through francium. Comparing with the results of Table II, we see corresponding theoretical contributions of 12 ppm, 39ppm, 149ppm, 488ppm, 606ppm, and 2158 ppm. Thus these contributions are in principle measurable for several of the alkalis, particularly cesium and francium.

For the case of lithium, the present approach is certainly less accurate than that available from nonrelativistic quantum electrodynamics (NRQED). The nonrelativistic lithium wavefunction is extremely well understood using variational techniques [16], and the same kind of NRQED calculations reported in Ref. [9] for hydrogenic ions can certainly be used for neutral lithium. A nonrelativistic evaluation of the lifetime of the lithium $2p_{1/2}$ state has been carried out in Ref. [20] with a result of 27.1045(14) ns which includes finite mass corrections along with relativistic corrections. The quoted error came mainly from the fact that the relativistic corrections were not directly calculated, and could be eliminated with rigorous relativistic calculations. Regardless, the radiative correction calculated here is so small that it should not make any difference to the theoretical lifetime in comparing with experiment.

The cases of cesium and francium are more problematic because of the complexity of their wave functions. A great deal of effort has gone into treating these wave function accurately, largely spurred by interest in PNC transitions. However, while experimental accuracy is now certainly high enough for detection of radiative corrections of the size found here for cesium and almost at the level needed for francium, further advances in many-body theory will be required, as was the case for the Lamb shift [14] and the hyperfine splitting [15], before one can decisively say this effect has been seen. Incidentally, a theoretical advantage for lifetimes as compared to hyperfine splittings is the relative insensitivity of lifetimes to nuclear effects: as emphasized in Ref. [15], uncertainties in the distribution of nuclear magnetism lead to theoretical uncertainties that are difficult to control.

A major spur to theoretical work on accurate cesium wave functions is the fact that the observation of PNC in the atom [10] has significant implications for particle physics. Before the large binding correction to the lowest-order radiative correction $-\alpha/2\pi$ was found, a discrepancy with experiment existed. As discussed in the introduction, a motivation for the present work was to provide a basis for the full calculation of the diagrams shown in Fig. 1. We note, for example, that diagram 1c is of the same form as the vertex diagram V in Fig. 2b, with the difference that the $6s$ (or $6p_{1/2}$) state in diagram 2b must be replaced with a perturbed orbital of the same parity which arises from the $6p_{1/2}$ (or $6s$) state perturbed by a Z boson exchange with the nucleus. Thus a relatively straightforward modification of the codes developed for the present calculations will allow the determination of this term. In fact, with the exception of diagram 1e, the entire PNC calculations of Fig. 1 can be

treated as perturbations of either the weak interaction calculations [13], the self-energy calculations [14], or the present transition matrix calculations. We are thus in a position to carry out the bulk of the full radiative correction calculations. The fact that the previously neglected radiative correction to the E1 matrix element is $0.261\alpha/\pi$ as shown in Table II makes it likely that the full PNC calculation will differ quantitatively, though perhaps not qualitatively, from the calculations that consider only the Z vertex.

Since the lowest-order results presented in Table I are substantially corrected by higher-order effects in many-body perturbation theory (MBPT) [21], similar corrections can be expected for the radiative corrections. The simplest way to account for these corrections is to assume that R_{wv} , defined as a ratio of the radiative correction to the lowest-order matrix element, is valid when the lowest-order dipole matrix element is replaced with more accurate results obtained with MBPT methods. As the size of the radiative corrections is small, even if this is only roughly true, the basic size of the effect would have been established here. To attain more accuracy, a QED perturbation theory approach could be taken. In that case, one would first consider graphs of the type shown in Fig. 1 where the Z vertex is replaced by interactions with other electrons. For even higher accuracy, one could consider yet more complicated graphs with one absorbed photon, one radiative photon, and two interactions with the other electrons: this is known, in the case without the radiative photon, to give results within a few percent of the experimental answers [21]. Progress in these large scale calculations, taken together with continuing advances in experiment and many-body methods, should allow tests of the lifetimes of the alkalis at the level achieved for positronium.

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TABLE I: Dipole matrix elements r_{wv} for $ns_{1/2} - np_{1/2}$ and $ns_{1/2} - np_{3/2}$ transitions in the alkalis with Kohn-Sham potentials. Units a.u..

transition	Li	Na	K	Rb	Cs	Fr
$ns_{1/2} - np_{1/2}$	4.171	4.588	5.681	6.009	6.585	6.511
$ns_{1/2} - np_{3/2}$	4.171	4.587	5.679	5.995	6.545	6.328

TABLE II: Self-energy contributions R_{wv} to E1 matrix elements for $ns_{1/2} - np_{1/2}$ transitions in the alkalis: error of 0.001 for Sum. Units $(\alpha/\pi)z_{wv}$.

Term	Li	Na	K	Rb	Cs	Fr
$R_{wv}(V; 00)$	-9.162	-9.308	-9.509	-9.573	-9.655	-9.647
$R_{wv}(V; iw)$	134.712	105.062	127.516	125.136	132.719	112.464
$R_{wv}(V; \text{Poles})$	-137.256	-107.481	-129.741	-127.273	-134.877	-114.624
$R_{wv}(D)$	11.698	11.712	11.731	11.738	11.748	11.747
$R_{wv}(\text{PO}; ns)$	0.003	0.031	0.067	0.182	0.326	0.787
$R_{wv}(\text{PO}; np)$	0.000	0.001	0.000	0.000	0.000	0.202
Sum	-0.005	0.017	0.064	0.210	0.261	0.929

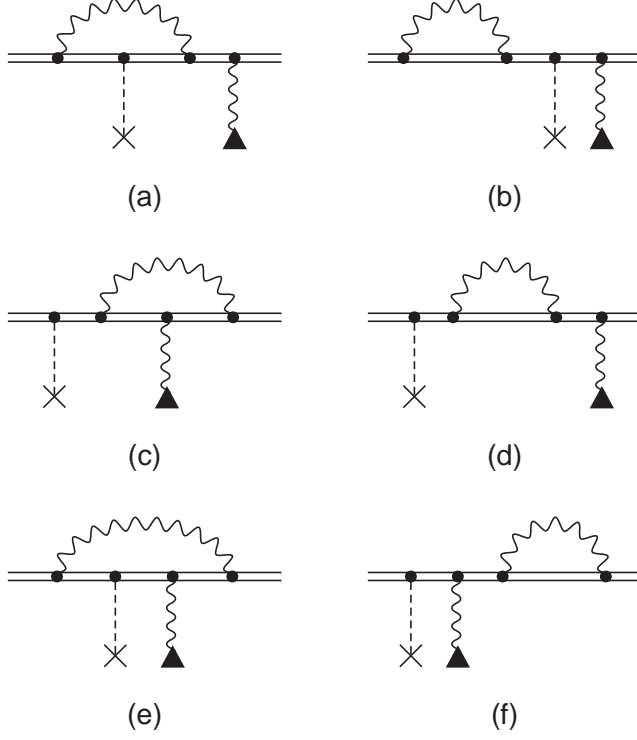


FIG. 1: Feynman diagrams for the radiative correction to electron excitation by a laser photon, indicated by the wavy line terminated with a triangle, in the presence of interaction with the nucleus through the exchange of a Z boson, indicated by the dashed line terminated with a cross.

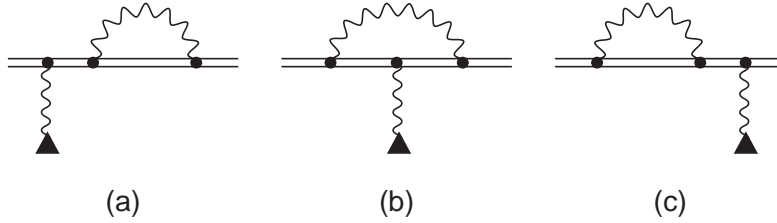


FIG. 2: Feynman diagrams for the radiative correction to the matrix element for $ns + \text{photon} \rightarrow np$.